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AN ANALYTICAL COMPARISON OF RANDOM AND EXHAUSTIVE SEARCH OF AN EXPANDING AREA WITH BINARY SENSORS

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ABSTRACT

In this study we analyze the problem of searching an expanding area over time with binary sensors. This problem can be applied to scenarios where the searcher has the location information of a mobile target with a time delay and the target speed is known. We consider two basic search models, random and exhaustive, and analyze formulas to measure the effectiveness of search process in terms of cumulative detection probability and compare the results for both plans. We also derive analytical expressions that will assist the decision maker in planning and utilizing his/her search effort. The results are verified with Monte Carlo simulations.

Keywords: Expanding area search, mobile target, search theory

GENİŞLEYEN BİR SAHANIN KESİN MENZİLLİ SENSÖRLER İLE GELİŞİGÜZEL VE TAM ARANMASININ ANALİTİK OLARAK KARŞILAŞTIRILMASI

ÖZET

Bu çalışmada zaman ile birlikte genişleyen bir sahanın kesin menzilli sensörler ile aranması problemi incelenmiştir. Söz konusu problem bir hedefin süratinin ve zaman gecikmeli olarak mevkisinin bilindiği problemlere uygulanabilir. Çalışma kapsamında gelişigüzel ve tam olmak üzere iki tip arama modeli ele alınmış ve etkinliklerini birikimli tespit olasılığıyla ölçen formüller incelenmiştir. Ayrıca planlayıcıya arama gayretini planlama ve kullanmada destek sağlamak üzere faydalı analitik sonuçlar da elde edilmiştir. Elde edilen sonuçlar Monte Carlo simülasyonları ile doğrulanmıştır.

Anahtar Kelimeler: Genişleyen saha araması, hareketli hedef, arama teorisi

1. INTRODUCTION

Search theory which basically involves the problems related with finding an object of interest that is missing or lost has numerous applications in diverse fields for decades. The mathematical basis of search theory has been a subject of serious study since World War II. (Koopman, 1946, 1980) established the basis for a rigorous study of search theory and practice with his pioneering work during World War II (Cooper, 2003). Since then search theory played an important role in military and non-military applications. It is used extensively for such things as mineral deposits, Search and Rescue (SAR) operations, police operations, pattern recognition, disease or contamination, medical diagnostics, and markets (Koopman, 1999). Some military applications involving search operations include searching for drug interdiction, broad area searches with ships (maritime problems), aircrafts or satellites, hunting for mobile missile launchers, terrorist combat groups, smugglers, etc.

The above applications can be categorized into three main types of search problems; barrier patrol, point coverage, and area search, each of which is geared towards a specific purpose (Cardie and Wu, 2004). In a barrier patrol, the adversary (or the target) has a specific goal of approaching a port, base or high value platform. To prevent the adversary to accomplish its mission the searcher must deploy sensors such that the probability of detection is maximized. Instead of establishing a traversal towards a single place, in point coverage scenarios, multiple points of interest must be protected. Therefore, sensors are deployed in the vicinity of these points of interest, while the remainder of the area remains relatively void of sensors. Area search scenarios consist of a general area of the ocean being covered by a field of sensors (Cardie and Wu, 2004; Golen, 2009). In an area clearance scenario, there are no obvious points of interests that an adversary would gravitate towards, such as a port. Thus, the field designer is left to make an intelligent guess as to how to optimally allocate the limited amount of search effort. Clearly, area clearance scenarios are relatively more complex than barrier and point coverage scenarios. There has been plenty of work in the literature for all these types of problems and an excellent survey of search theory literature is available in (Benkoski, et al., 1991). Besides early analytical works, evolutionary algorithms plays an important role in search theory especially for large scale search and sensor optimization problems. For such problems, evolutionary algorithms were first introduced to the field by (Holland, 1975). Some other significant work are; (Raisanen and Whitaker,2003) for optimizing cell coverage in cellular networks, (Xue et al., 2003) for minimizing the power consumption of a wireless network, (Ranganathan et al., 2006) for optimizing communication paths in heterogeneous sensor networks, (Barrett, 2007), (Jourdan and Weck, 2004), (Spanache et al., 2004) for the applications of genetic algorithm for optimization sensor deployment problems. Studies in search theory involving traditional monostatic sensors are followed by multistatic sensor network studies mainly on sensor & ping optimization and multistatic tracking algorithm categories. In their study (DelBalzo et al., 2005), analyze the oceanographic effects on such systems. (Washburn, 2010) and (Walsh and Wettergren, 2008) approximate the search performance for multistatic fields. (Karataş, 2012), (Tharmarasa, et al., 2009), (Patrick, 2006) analyze the problem of determining the optimal number and placement of multistatic sonar sensors to achieve maximal coverage. (David et al., 2009), (Saksena and Wang, 2008), (Wang et al., 2008) analyze the problem of ping scheduling and strategies while optimizing both the temporal and spatial sensing coverage. (Erdinc,2006), (El-Jaber et al., 2009), (Ehlers et al., 2009), (Orlando et al., 2010), (Anastasio et al., 2010), (Daun and Ehlers, 2010), (Orlando and Ehlers, 2011) work on the tracking algorithms for multistatic systems.

A logical extension of area search problems involves searching an expanding area over time with traditional monostatic sensors which may be later on expanded to the multistatic case. As opposed to searching a fixed area, in an expanding area search problem the area to be searched does not have stable borders and a specific shape. This type of problems can be applied to scenarios where the searcher has the location information of a mobile target with a time delay and the target speed is known. Real life problems in SAR operations such as searching for a life raft adrift in the ocean, searching for the survivors from a plane crash or the crewmembers of a sunken boat as well as searching adversaries with datum information that are trying to evade the searcher are a few examples of expanding area search problems. In SAR operations maximizing the *CDP* at the greatest possible rate with the available search resources will save more lives by finding and assisting persons in need more quickly. Time is an important factor in saving lives, especially when the distressed person is injured or the weather has become extreme enough to threaten continued survival (Cooper et al., 2003). There are analytical methods and formulas developed for measuring the effectiveness - Cumulative Detection Probability (*CDP*) - of area search where you apply random or exhaustive methods for a fixed area. Solutions to such problems as in (Koopman,1946, 1980, 1853) and (Washburn, 1981, 1983) in the literature assume that search effort is an abstract quantity that may be allocated arbitrarily over the search space, and the CDP is a function of total amount of effort applied to target's location (Kierstead, 2003). However, facing an expanding area search problem, the formulas derived for fixed area search problems do not express the planner needs to evaluate the expected effectiveness of his search effort. This extension emerges the need to develop analytical models and scientific arguments for such problems.

This article addresses and analyzes a basic problem in search theory concerning the possible future distribution of a target's location. Given an initial target location and a constant velocity, what is the chance of finding it with a binary sensor? In specific, we focus on the following actual area search problem: there is a searcher and a target where the objective of the searcher is to detect the target. The objective of the target may vary depending on the scenario, such that an adversary target may try to stay undetected and act non-cooperatively whereas a neutral target of interest may be moving subject to drift, wind, etc. The searcher receives the location information of the target for a very short period of time and looses it. Just after getting the location information the searcher approaches to the last known position (datum) of the target and the target starts moving away from datum. The area of target uncertainty increases within time. The Measure of Effectiveness (*MoE*) (or the objective function) of the search problem is the *CDP* at time *t*. We are analyzing analytical solutions for *MoE* when one or multiple searchers executes random and exhaustive search. This article contributes search theory literature in terms of analyzing the effectiveness of two basic search strategies, namely random and exhaustive, for searching an expanding area over time. Deriving analytical expressions to compare both strategies, our main ambition is to assist decision makers in planning, utilizing and conducting search effort as well as depicting the cause-effect relationship of this special case of search process using crucial parameters like time delay, searcher capability and target speed.

The organization of the paper is as follows. First, general definitions related to the search theory are defined in Section 2. Section 3 presents the formal problem statement for the expanding area search problem. Section 4 gives an overview to random and exhaustive search models. Section 4 proposes the expanding area search model and analytical results for random and exhaustive search. Comparison of our analytical estimates with Monte Carlo simulation data is presented in section 6 and finally section 7 summarizes the main results.

2. DEFINITIONS

In this section are given some useful definitions related to the search theory that form a basis to the computations.

2.1 Search

Search process is an operation that uses available resources to find persons or objects of interest whose exact location is currently unknown. The formal description of search is "*to go or look through (a place, area, etc.) carefully in order to find something missing or lost*" (Oxford Dictionary, 2010). (Koopman, 1999) describes the operation of search as "an organic whole having a structure of its own—more than the sum of its parts."

2.2 Searcher and Target

The "target" as the object of interest while the "searcher" is the object or agent concerned with finding the target.

2.3 Last Known Position (Datum)

The last witnessed, reported or substantiated (by clues or evidence) location of the missing/lost object/target. In search theory generally the term "Datum" is used to describe any point on a map which conveys information to the person reading it (Skip and Brett, 2008).

2.4 Probability of Detection (*PoD*)

It is the conditional probability that the target will be detected during a single sortie if the target is present in the area searched during the sortie (Champagne et al., 2003).

2.5 Cumulative Detection Probability (*CDP*)

Cumulative detection probability is the cumulative probability of detecting (*PoD*) the target given that it was in the searched area on each of several successive searches of that area (Champagne et al., 2003). The probability that at least one detection no later than time *t* is expressed as $F_T(t)$.

2.6 Binary Sensor (Cookie Cutter)

It is common to approximate a searcher's the average ability to detect a particular target under a specific set of environmental conditions in order to facilitate analytical modeling of the search process. The simplest approximation is that of a definite range law: targets which come within a certain distance of the searcher are always detected, and targets which do not come that close are never detected (hence the searcher cuts a clean swath like a "cookie cutter") (Wagner et al., 1999). This model thus yields the correct number of targets detected by a single searcher making a single pass through an area. Also, the sensor is characterized by only a single parameter: its maximum detection range, R. Binary sensors are also known as cookie cutter sensors with a sweep width of $w=2R$ defined as follows:

$$PoD(x) = \begin{cases} 1, & x \in [-R, R] \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where x is the distance between the searcher and target.

2.7 Coverage Ratio (*C*)

Coverage factor is a relative measure of how thoroughly an area has been searched, or "covered." It is defined as the ratio of the area effectively swept to the physical area of the segment that was searched (Cooper et al., 2003). If we are searching an area of size A for a period of time t with a searcher of speed v and sweep width of w, then coverage ratio is:

$$C = \frac{\text{Area Effectively Swept}}{\text{Search Area}} = \frac{vwt}{A} \tag{2}$$

2.8 Detection Rate

Detection rate is commonly referred as the *PoD* per unit time and if the rate is not constant, i.e. non-homogeneous within time, it is shown as $\gamma(t)$, the detection rate at time t. For homogeneous environments detection rate is a constant, λ.

3. PROBLEM STATEMENT

The search problem analyzed in this study can be described in general as a "two-sided search problem where a continuous random or exhaustive search effort is applied by one or more search units to detect a moving target in a continuous expanding target space" (see Figure 1).

The simplest types of search problems are one-sided search problems in which the searcher can choose his strategy, but the search object neither chooses a strategy nor reacts to the search in any way. In two-sided search problems, both the search object and the searcher are allowed to choose their strategies.

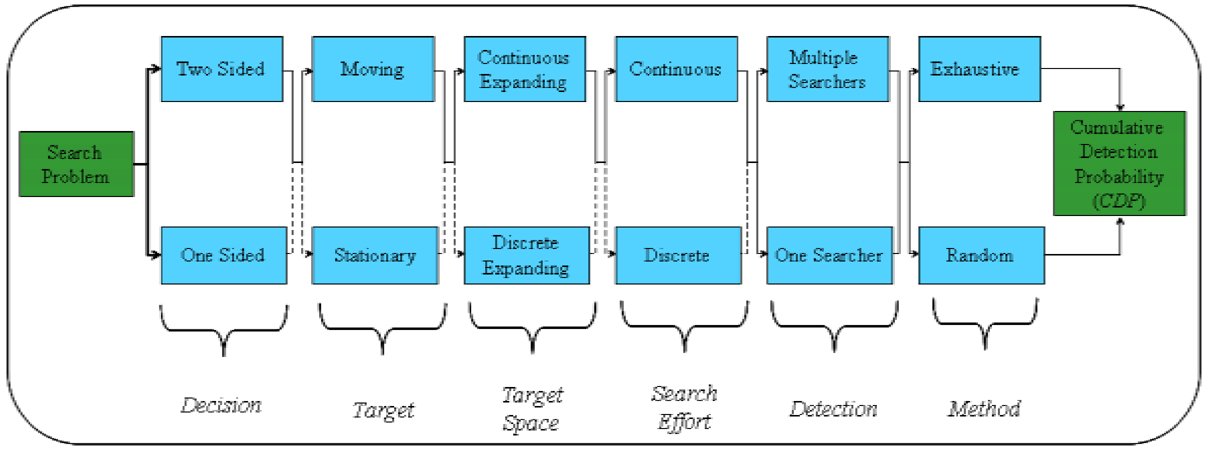

Figure 1. Search Problem Classification

The problem we are dealing with in this study can be regarded as both one and two sided problem since we assume that our search effectiveness prediction will be independent of the target motion model. In specific our model will consider the worst case situation by assuming that the area of uncertainty for the target is determined by the maximum distance it can travel during time *t*. The target may be moving in order not to be detected (non-cooperatively) or drifting with the wind and waves. The search effort is defined by the amount of time spent during a search. Since time is continuous, the search effort is continuous. The target space is taken as the expanding area of target uncertainty in which the target is expected to be present. Therefore, the target space is a continuous region. The effectiveness of the search is measured for two methods of area search, one for random and the other for exhaustive search.

In specific, the objective of the searcher is to detect the target and the mobile target is assumed to be neutral or non-cooperative. The searcher receives the location information of the target for a very short period of time and looses it. Just after getting the location information the searcher approaches to the last known position (datum) of the target and the target starts moving away from datum. The area of target uncertainty increases within time *t*. We are seeking analytical solutions for *CDP* when one or multiple searchers executes random and exhaustive search.

4. RANDOM AND EXHAUSTIVE SEARCH MODELS

4.1 Random Search Model

"Random search" as a concept plays a central role in search theory as its use places a lower bound on the probability of detection. It is a method where every unit of search effort is employed in an unsystematic manner – randomly- in the field of interest. Since it has been shown to follow an exponential distribution, random search is characterized by the "memoryless property" of that distribution (Washburn, 2002). This is consistent with intuition in that the length of time a searcher has been looking to detect a target has no bearing on subsequent detection probability.

4.1.1 Random Search for One Searcher

The main assumption is that the searcher is a cookie-cutter sensor with range *R* or sweep width of $w=2R$. In short intervals of Δt the increment of area searched is $a=wv\Delta t$. The searcher track consists of disjoint segments and each segment is uniformly & independently distributed in the search area *A*. (i.e., any point in *A* equally likely to be searched at any instant in time). Because the increment of area searched $a=wv\Delta t$, is uniformly distributed over *A*,

$$P\{\text{detection in } (t,t+\Delta t)\} = \frac{wv\Delta t}{A} \quad (3)$$

If we call $\gamma(t)$ = detection rate = P(detection)/unit time, the cumulative detection probability for a random search, CDP_{rand} with detection rate $\gamma(t)$ is (Stone, 1989; Koopman, 1980):

$$F_{T(rand)}(t) = 1-\exp\left(-\int_0^t \gamma(s)\,ds\right) \quad (4)$$

where

$$\gamma(t) = \frac{P\{\text{detection in } (t,t+\Delta t)\}}{\Delta t} = \frac{Wv\Delta t / A}{\Delta t} = \frac{Wv}{A} \quad (5)$$

And, with constant detection rate $\lambda=wv/A$, the CDP_{rand} can be restated as:

$$F_{T(rand)}(t) = 1-e^{-\lambda t} = 1-\exp\left(-\frac{wvt}{A}\right) \quad (6)$$

Hence substituting *C* from (2),

$$F_{T(rand)}(t) = 1-e^{-C} \quad (7)$$

4.1.2 Random Search for Multiple Searchers

The random search model generalizes for multiple searchers. If one searcher has detection rate λ, then *n* searchers have detection rate $n\lambda$ (Benkoski et al., 1991). Therefore CDP_{rand} for *n* searchers can be approximated as:

$$F_{T(rand)}(t) = 1-e^{-\lambda t} = 1-\exp\left(-n\frac{wvt}{A}\right) \quad (8)$$

4.2 Exhaustive Search Model

Exhaustive search is an area search model widely used in search theory. That is, the target is hiding somewhere in an area *A* and a perfect search is conducted by the searcher. There is no search overlap (every point in *A* is searched once before any point is searched twice), no search is conducted outside area *A* and all of area *A* is covered by the searcher sensor. (see Figure 2) in real life, actual area search, when trying to approximate the exhaustive search ideal, is usually conducted using parallel sweeps (mowing the lawn), spiral-in, or spiral-out paths (Wagner et al., 2004). Once again, the main assumption is that the searcher is a cookie-cutter sensor with a maximum range *R*.

4.2.1 Exhaustive Search for One Searcher

Since the search effort is expended ideally (with no overlap and no wasted effort), the cumulative detection probability for a exhaustive search, CDP_{exh} with one searcher is equal to the coverage factor for $C\le 1$, and equal to 1 for $C>1$ (Stone, 1989; Koopman, 1980).

$$F_{T(exh)}(t) = \begin{cases} \dfrac{wvt}{A} & ,C\le 1 \\ 1 & ,C>1 \end{cases} \quad (9)$$

or

$$F_{T(exh)}(t) = \min\left\{\frac{wvt}{A},1\right\} \quad (10)$$

4.2.2 Exhaustive Search for Multiple Searchers

The exhaustive search model also generalizes for multiple searchers. If multiple equivalent searchers are used to search the area *A*, the coverage is nC. Therefore CDP_{exh} for *n* searchers can be approximated as:

$$F_{T(exh)}(t) = \min\left\{n\frac{wvt}{A},1\right\} \quad (11)$$

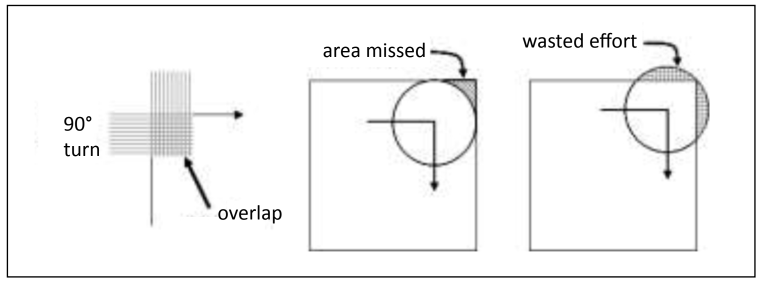

Figure 2. Overlap, Area Missed and Wasted Effort Examples

4.3 Comparison of Random and Exhaustive Searches for Fixed Area Search:

In exhaustive search the CDP_{exh} is equal to the coverage factor since the effort is expended ideally to the area. In random search the CDP_{rand} equals to "1-exp(-coverage factor)" since the effort is expended with wasteful overlap to the area. Figure 3 illustrates the *CDP* functions for both random and exhaustive searches executed in fixed areas (i.e. not expanding within time).

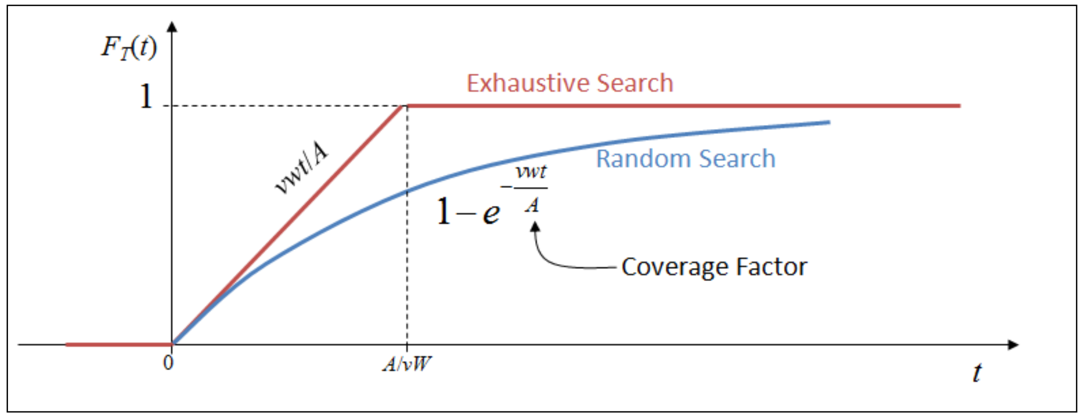

Figure 3. Exhaustive and Random Search *CDP* Comparison For a Fixed Area *A*

Being a completely haphazard and disorganized search model, random search does worse than exhaustive search because of wasteful overlap. A searcher could do worse than random search deliberately however random search is often considered a lower bound for search effort. Exhaustive search is an upper bound in search effectiveness (Washburn, 2002). It is better than what can be achieved for real search. Real search generally falls in between.

5. MODELING THE EXPANDING AREA SEARCH PROBLEM WITH BINARY SENSORS

In our problem after the searcher gets the datum information it immediately proceeds to the target uncertainty area and arrives at time $t=0$ to commence search at speed v. Target position (datum) was known t_0 time units before searcher arrives (time late). Target is evading or drifting at constant speed u in any direction which can change at any time. Therefore, at $t=0$ target is located somewhere in a circle, centered at datum, of radius $r_0=ut_0$. (See Figure 4) The searcher is a binary sensor with a maximum detection range of R and sweep width of $w=2R$.

The radius of the search area is initially r_0 and after t hours of fruitless search the target is assumed to be within a circle of radius r_0+ut and the radius grows by

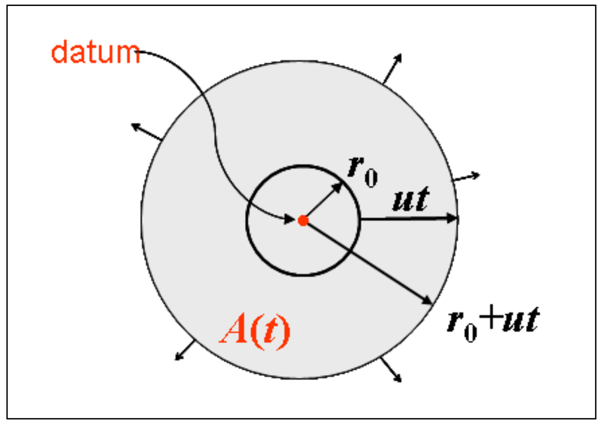

Figure 4. Expanding Area Search Problem

ut units in time *t*. In those search situations the target uncertainty area is an expanding localization circle whose area $A(t)$, after search time, t, is:

$$A(t)=\pi\left(r_0+ut\right)^2 \tag{12}$$

And substituting equation (5) in (12), the detection rate is:

$$\gamma(t)=\frac{wv}{\pi\left(r_0+ut\right)^2} \tag{13}$$

5.1 Random Search of an Expanding Area

For the random search problem we use the general equation (4) for computing the CDP_{rand}. By using the detection rate (13), (Coggins, 1971) and (Washburn, 1980) express the CDP_{rand} in an expanding area as:

$$F_{T(rand)}(t)=1-\exp\left(-\int_0^t \frac{wv}{\pi\,(r_0+u\tau)^2}\,d\tau\right) \tag{14}$$

Evaluating the integral in the exponent we get,

$$\int_0^t \frac{wv}{\pi\,(r_0+u\tau)^2}\,d\tau=\left(\frac{wv}{\pi u^2 t_0}\right)\left(\frac{t}{t_0+t}\right) \tag{15}$$

Therefore *CDP* is;

$$F_{T(rand)}(t)=1-\exp\left(-\frac{wv}{\pi\,u^2 t_0}\,\frac{t}{(t_0+t)}\right)\ \text{for all } t>0 \tag{16}$$

As t gets large, $t/(t_0+t)\to 1$, hence the maximum CDP_{rand} that can be reached is,

$$CDP_{rand}^{\max}=F_{T(rand)}(\infty)=1-\exp\left(-\frac{wv}{\pi u^2 t_0}\right) \tag{17}$$

5.2 Exhaustive Search of an Expanding Area

If we are executing an exhaustive search, the same rule in (10) applies to CDP_{exh} in expanding area case too where the *CDP* equals to the coverage factor *C*. Therefore we can simply compute;

$$F_{T(exh)}(t)=\min\left\{\frac{wv}{\pi\,u^2 t_0}\,\frac{t}{(t_0+t)},1\right\} \tag{18}$$

Similarly, as t gets large, the maximum CDP_{exh} that can be reached is,

$$CDP_{exh}^{\max}=F_{T(exh)}(\infty)=\min\left\{\frac{wv}{\pi\,u^2 t_0},1\right\} \tag{19}$$

For multiple searchers it is easy to show that *CDP* is computed by multiplying the expressions in parenthesis with the number of searchers, *n* for both random and exhaustive search cases.

5.3 Additional Analytical Results

For random search, if we call t_α the time to achieve some fractional amount, α, of $CDP_{rand}^{\max}$, for $0\le\alpha\le 1$, then

$$\alpha=\frac{F_{T(rand)}(t_\alpha)}{CDP_{rand}^{\max}} \tag{20}$$

and

$$\alpha=\frac{1-\exp\left(-\frac{wv}{\pi u^2 t_0}\,\frac{t_\alpha}{(t_0+t_\alpha)}\right)}{1-\exp\left(-\frac{wv}{\pi u^2 t_0}\right)} \tag{21}$$

solves

$$t_\alpha=\frac{wv/\pi u^2}{\ln\left[(1-\alpha)\exp\left(wv/\pi u^2 t_0\right)+\alpha\right]}-t_0 \tag{22}$$

For exhaustive search, if we call t_α the time to achieve some fractional amount, α, of $CDP_{exh}^{\max}$, for, $0\le\alpha\le 1$ and if $CDP_{exh}^{\max}<1.0$ then

$$\alpha=\frac{\frac{wv}{\pi u^2 t_0}\,\frac{t_\alpha}{(t_0+t_\alpha)}}{\frac{wv}{\pi u^2 t_0}} \tag{23}$$

solves

$$t_\alpha=t_0\cdot\frac{\alpha}{1-\alpha}\qquad(\text{for } \alpha\neq 1) \tag{24}$$

For example; the searcher with a sweep width of $w=1$ nm (nautical mile) starts the search 30 min. after getting the datum information. The target evades with a speed of $u=10$ nm/hr and the searcher speed is $v=250$ nm/hr. *CDP*s for random and exhaustive search are computed by equations (16) and (18) and maximum *CDP*s are computed by equations (17) and (19) respectively. The plot of *CDP*s with respect to search time *t* is in Figure 5. As seen from the plot, the maximum *CDP* for exhaustive search is 1.0 whereas

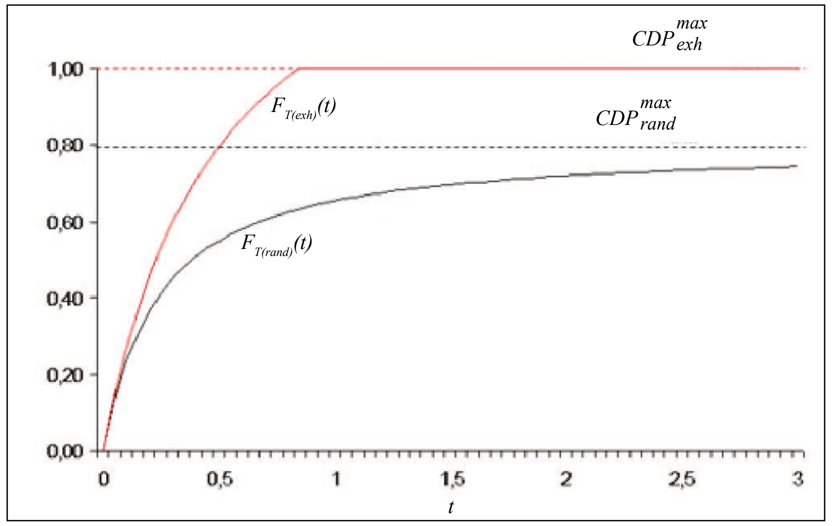

Figure 5. *CDP* Plots For Random and Exhaustive Searches

with the random search the probability goes down to 0.8 allowing a 0.2 chance of escape to the target.

Also by using equation (22) the effect of "knee of the curve" or "diminishing returns" for random search can easily be seen in this example by computing that α=82% of maximum *CDP* is attained after $t_\alpha=1$ hr of search, α=90% of maximum *CDP* after $t_\alpha=2$ hrs and α=93% of maximum *CDP* after $t_\alpha=3$ hrs. Almost an infinite amount of time is needed to get the remaining %7.

Figure 6 illustrates the *CDP* plots for random and exhaustive searches for t_0 delay times 0.25, 0.50, 0.75 and 1.0 hr. The plots show that exhaustive search always performs better than the random search for expanding area search problems as in fixed area search problems. However for small *t* the difference between two is negligible when compared to the large values of *t*. Another result is that as the delay time t_0 increases the *CDP* decreases for both search plans which means that if the searcher starts the search process soon after the datum time (with very small t_0) it is highly probable to detect the target of interest.

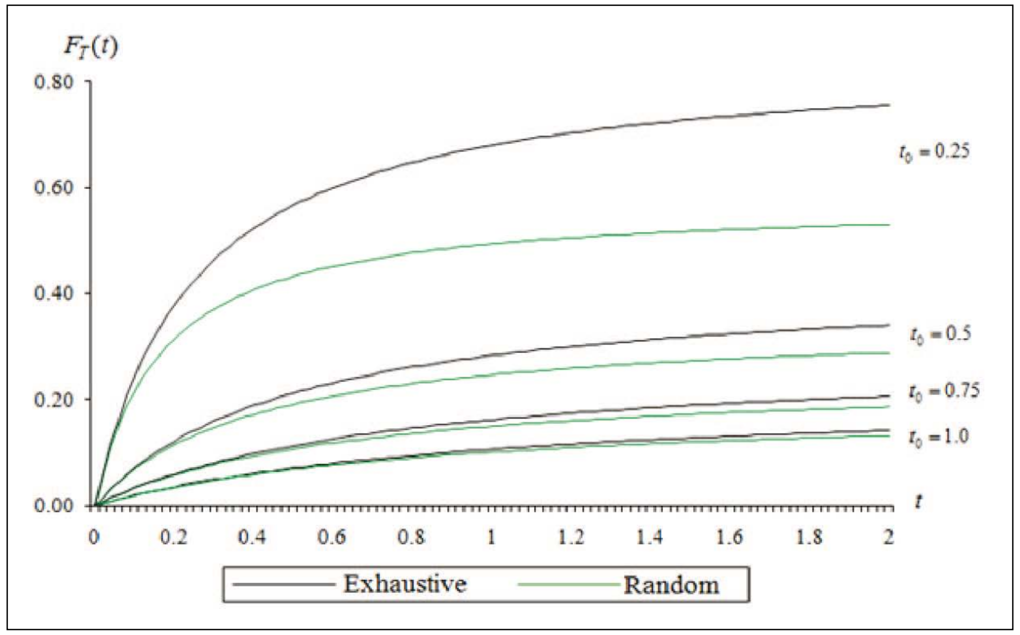

Figure 6. *CDP* Plots For Random and Exhaustive Searches For t_0=0.25; 0.50; 0.75; 1.0 and n=1

6. SIMULATION

In order to verify the theoretical results derived in this study we test both the random and exhaustive search models with Monte Carlo simulations in MATLAB®. For each simulation run we generate $k=10^4$ targets which move with a fixed speed $u=15$ nm/hr away from the datum to a random direction. The search effort is discretized by allocating it at every small time interval $\Delta t=0.02$ hr. where we compute its size by $a=wv\Delta t$. The simulations are repeated for different values of time delay as t_0=0.25; 0.50; 0.75; 1.0. For random search the simulations are repeated for number of searchers n=1, 2 and 3. The searcher speed is set to $v=30$ nm/hr and sweep width $w=5$ nm constant. For each parameter set we generate $i=10^3$ iterations and we repeat all parameter sets for random and exhaustive search plans until time $T_{max}=2$ hrs.

Table 1: Simulation Parameter Sets

Search Plan	Time delay (hr)	Searcher speed (km/hr)	Target speed (km/hr)	Sweep width (km)	# of searchers	Time step (hr)	Sim. stop time (hr)	# of targets	# of iterations per Run
	t_0	v	u	w	n	Δt	T_{max}	k	i
Random	0.25; 0.50 0.75; 1.0	30	15	5	1; 2; 3	0.02	2	10^4	10^3
Exhaustive	0.25; 0.50 0.75; 1.0				1				

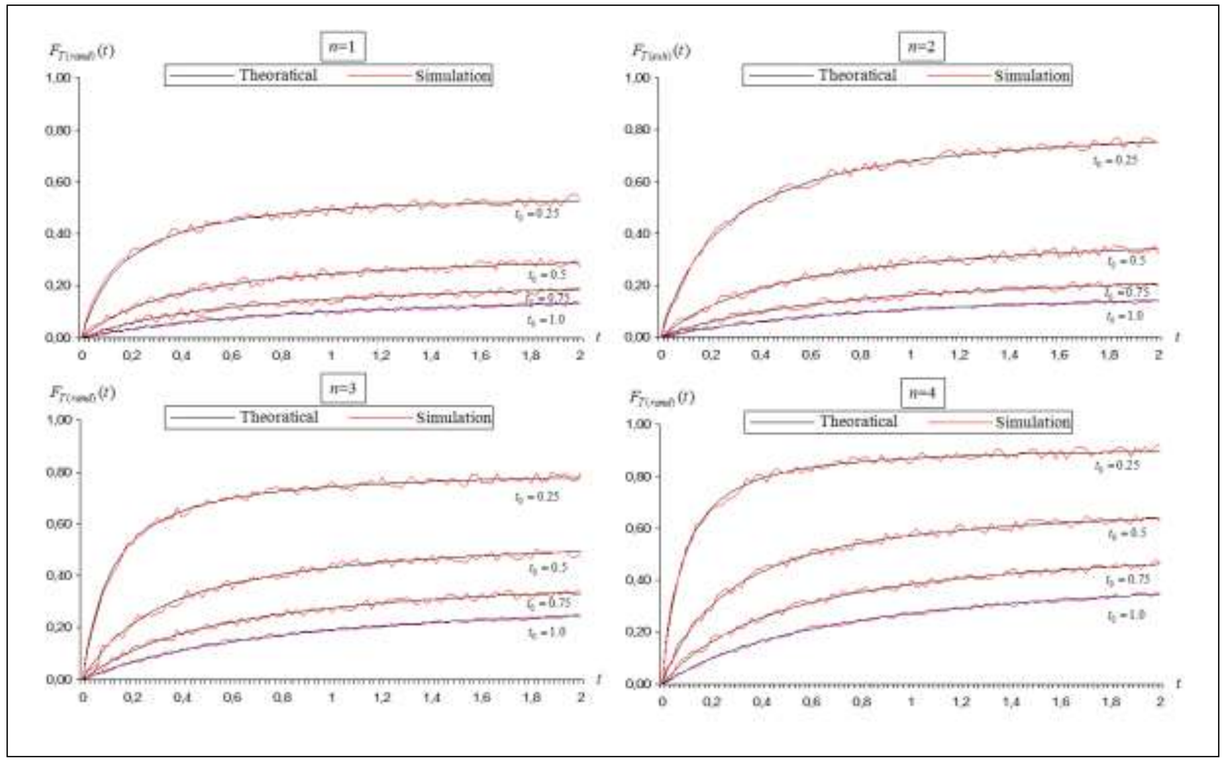

Figure 7. Simulation Results For Random Search For t_0=0.25; 0.50; 0.75; 1.0 (n=1, 2, 3 and 4)

The summary of parameter sets for simulation runs are as in Table 1.

After running the simulations it is observed that the results are very close to the theoretical results derived in previous section. As the delay time t_0 increases, *CDP* for any value of time decreases as well as the maximum *CDP* that can be achieved. As the number of searchers, *n*, increase the search effectiveness increases but this increase is not linear with *n* for random search. The comparison of simulation results with the theoretical values are summarized in Figure 7 below. It is also obvious that due to the diminishing returns effect it would be a good idea to consider stopping spending more search effort after a certain value of *t*. For example, in the random search case with $n=2$ and $t_0=0.25$ after searching the area for $t=0.3$ hr the *CDP* is 0.6, however after searching for another 0.3 hr. the updated *CDP* is only 0.7, with an increment of only 0.1. So the planners may consider stopping to spend search effort after 0.3 hr because of the diminishing returns effect.

7. SUMMARY

In this study we analyze the search problem where we search for a target with an area of uncertainty that is increasing over time and we have the options of conducting random or exhaustive search. Modeling this specific search problem we derive the effectiveness of the search in terms of Cumulative Detection Probability (*CDP*) using basic parameters such as searcher and target speed, sweep width (searcher capability) and search delay time. To come up with closed form results we use the well-known search theory formulas for random and exhaustive searches. We also derive useful analytical results that will assist decision makers in planning and utilizing search effort considering the percentage of maximum *CDP* that can be achieved. Using formulae (16) and (18) it is possible to approximate the *CDP* for random and exhaustive searches respectively. Formulae (22) and (24) can be used to approximate the amount of time needed to achieve a certain fraction of maximum *CDP* for searching expanding areas with random and exhaustive search respectively. The analytical results are verified with Monte Carlo simulations. The results seem to be in accordance with the theory. In specific the results derived in this study can be applied to military operations to search mobile evaders or search and rescue operations to search for targets such as a life raft adrift in the ocean.

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